The Development of an Instrument to Measure Geometric Creativity

Mohamed El-Demerdash
University of Education Schwäbisch Gmünd
m_eldemerdash70@yahoo.com

Ulrich Kortenkamp
University of Education Schwäbisch Gmünd
kortenkamp@cinderella.de

Abstract

There is no doubt that students can learn and develop their creative potential, if we use the appropriate programs that successfully teach them the creativity skills and its operations. However, in order to measure the effectiveness of such programs we need an instrument to assess creativity. This paper presents the development of a test that can be used to assess geometric creativity and to obtain concrete indicators of creative potential in geometry. The test was designed as a part of larger experimental study conducted to assess the geometric creativity among mathematically gifted students, and to develop their geometric creativity using dynamic geometry software. It uses four components that we consider to be basic ingredients of geometric creativity: fluency (based on the number relevant responses), flexibility (based on the number of different categories of the relevant responses), originality (based on the statistical infrequency of responses in relation to peer group), and elaboration (based on number of redefined follow-up questions or problems).

Keywords: geometric creativity, creativity, fluency, flexibility, originality, elaboration.

1 Introduction

Developing creativity has become a major topic discussed in many international conferences and meetings taking place in many countries. As a result of that many educational programs were developed to provide students with various experiences in order to promote their creative potential (e.g. Cho et al 2004, Mohamed, 2003, among others). In this concern, two main approaches appear. Some researchers see that creative potential can be learned and developed directly using specific programs that teach creativity skills and its operations regardless to the subject matters, while others assert that teaching creativity should be associated to the subject matters and it should be a part of the corresponding lesson plans that teachers prepare (Jerwan, 2002, p. 38).

With respect to mathematics, there has been a growing interest in using *mathematical content* to improve creativity and as a result new terminologies about creativity appear, such as *mathematical creativity,* which refers to creativity in the field of mathematics. Also, recent studies used geometric content to develop the creative potential among students (e.g. El-Rayashy & Ibrahim Al-Baz Mohamed 2000, Ibrahim Al-Baz Mohamed 1999, Mohamed, 2003).

Even though many mathematical programs were designed aiming at developing creativity, most of these programs do not provide a practical method to *assess* the creative potential in the field of mathematics or geometry in order to decide on the effectiveness of these programs in developing the students' creativity. This is a methodological problem – we cannot claim the success (or failure) of a program without being able to measure its effect.

The development of a geometric creativity test, which is presented in this paper, is a part of larger experimental study that aims at developing an enrichment program using dynamic geometry software and deciding on its effectiveness in enhancing the mathematically gifted students' geometric creativity in high school (El-Demerdash, in preparation). The development of this test came as a research necessity to obtain an instrument that copes with the purposes of the study, and as such is of interested on its

own. It can be used to assess geometric creativity in terms of its four components (fluency, flexibility, originality, and elaboration), which are considered in the study.

In this paper we describe the test development in detail and how it can be used to assess geometric creativity and to obtain concrete indicators of creative potential in the field of geometry.

2 Test Design

We used a 6-step process to design the geometric creativity test (short: GCT).

1. Specification of the Aim of the Test
2. Specification of the Components that the Test Measures
3. Creation of a Preliminary Form of the Test
4. Setup of a Grading Method for the Test
5. Content Validity Check
6. Test-Piloting

In the following sections we will elaborate on each step. For the final result of the design process we refer to the complete test that is available as a download from http://cinderella.de/material/gct.

2.1 Specification of the Aim of the Test

The aim of the geometric creativity test GCT is to obtain a validated and reliable instrument that can be used to assess geometric creativity and get concrete indicators of creative potential in the field of geometry. If possible, fine-grained information about the creativity components should be available.

2.2 Specification of the Components that the Test Measures

By reviewing literature and prior studies[1] related to the subject of creativity, mathematical creativity, and geometric creativity, we were able to determine the geometric creativity components to be measured by the test should measure as follows:

1. **Fluency:** the student's ability to pose or come up with many geometric ideas or configurations related to a geometric problem or situation in a short time.
2. **Flexibility:** the student's ability to vary the approach or suggest a variety of different methods toward a geometric problem or situation.
3. **Originality/Novelty:** the student's ability to try novel or unusual approaches toward a geometric problem or situation.
4. **Elaboration:** the student's ability to redefine a single geometric problem or the situation to create others, which are not the geometric problem situation itself or its solutions, but rather the careful thinking upon the particular aspects that govern the geometric problem or situation changing one or more of these aspects by substituting, combining, adapting, altering, expanding, eliminating,

[1] See El-Rayashy & Ibrahim Al-Baz Mohamed 2000, Haylock 1997, Ibrahim Al-Baz Mohamed 1999, Mann 2005, Mohamed 2003, Nakin 2003, Park 2004, Lee & Shim 2005.

rearranging, or reversing and then speculating on how this single change would have a ripple effect on other aspects of the problem or the situation at hand.

The combination of these four components defines our notion of geometric creativity.

2.3 Creation of a Preliminary Form of the Test

This step includes identifying test specifications, item types, writing items, and writing directions of the test. The preliminary form of the test is available from http://cinderella.de/materials/gkt/.

For each of the four components found in step 2, special items to test them were created. Table 1 shows the geometric creativity components of the test, the items that were designed to measure each component, the number of items corresponding to each component, and the percentage of each component.

Table 1

Specifications of the geometric creativity test

Components of the geometric creativity	Items	Number of items	Percentage
Fluency	1, 2, 3, 4	(4 items)	33%
Flexibility	5,6,9	(3 items)	25%
Originality/Novelty	7,8,12	(3 items)	25%
Elaboration	10,11	(2 items)	17%
Overall Geometric creativity test	1 to 12	(12 items)	100%

Concerning the item types in the test, the GCT includes open-ended and non-routine geometric situations and problems that require producing many various and different responses. In designing these situations and problems the researchers took into consideration some criteria for a task to be effective in revealing geometric creativity and in distinguishing between students in a particular population in terms of the creativity of their responses:

(1) The students' responses should show a wide range of geometric and mathematical ideas.

(2) A large number of appropriate responses are possible for these students.

(3) The students' responses should show a consistent interpretation of the instruction in the task.

(4) There should be several clear responses that can be obtained by most students.

(5) There should be a number of appropriate responses that are obtained by relatively few students.

(6) These original responses should have a degree of face validity for indicating creative ability in geometry and they should not be geometrically trivial (Haylock, 1997, p. 72).

Regarding the writing items of the test, they were written in verbal and nonverbal (symbols and figures) ways and a vision of the expected responses for each item of the test was put into account.

The GCT, in its preliminary form, consists of 12 items that are distributed among the four components of geometric creativity: fluency, flexibility, originality, and elaboration. Items 1, 2, 3, and 4 were designed to assess students' geometric fluency. Items 5, 6, and 9 were designed to allow students to come up with not only many ideas but also many categories of ideas in order to assess their geometric flexibility. Items 7, 8, and 12 were designed to allow students to show unusual and unique ways of solutions to find out how many original/novel geometric ideas they have. Items 10 and 11 were designed to assess students' geometric creativity in elaborating a geometric problem or situation.

Before we move on to the grading method used for the GCT we want to explain how each item, though designed for a certain component of creativity, is used to measure some other components at the same time. We use item 1 to explain how each item can be used to assess different components of geometric creativity. Item 1 – originally designed for testing fluency – requires a student to write down as many geometric concepts and terminologies as possible that starts with the letter p. Let us assume these responses: parallel, parallelogram, perpendicular, polyeder, pyramid, point, and point of symmetry as shown in Table 2.

Table 2

Item 1 and assessing different geometric creativity components

Student's Responses	Flu.	Flex.	Ori.
Parallel	1	C1	0
Parallelogram	1	C1	0
Perpendicular	1	C1	4
Polyeder	1	C2	1
Pyramid	1	C2	2
Point	1	C1	0
Point of symmetry	1	C3	3
Score	7	3	10

In this example, as the student comes up with 7 relevant responses, his/her fluency score on this item will be 7 points. Moreover, as the responses can be classified into three different categories according to different domains of geometry: Euclidean geometry, space geometry, and transformational geometry, which reflects the student's ability to vary his/her approach and break from mental sets to come up with not only different responses but also varied ones, therefore his/her flexibility score on this item will be 3 points. Similarly, the student's originality score can be assessed on this item, as the statistical infrequency of responses in relation to peer group responses. Each response will take zero, one, two, three or four points as an originality score based on its frequency. More details about grading the components will be given in the following section.

For completeness, we show in Table 3 which other components were assessed by the test items in addition to the ones they were designed for.

Table 3

Test items and geometric creativity components

	Fluency	Flexibility	Originality	Elaboration
Item 1	×	*	*	
Item 2	×	*	*	
Item 3	×	*	*	
Item 4	×	*	*	
Item 5	*	×	*	*
Item 6	*	×	*	*
Item 7	*	*	×	
Item 8	*	*	×	
Item 9	*	×	*	
Item 10	*	*	*	×
Item 11	*	*	*	×
Item 12	*	*	×	

× Indicates that the item was intentionally designed to assess this component.

* Indicates that the item would be used to assess this component and it was not intentionally designed to assess it.

As for the writing directions of the test, simple directions are written for the students, including some instructions that stimulate students' creative thinking. Instructions to inform students of the time allowed for the test and how to answer the test items were also included. The directions also indicate that the answer to each item is not restricted.

2.4 Setup of a Grading Method for the Test

Reviewing literature and prior studies[2] related to the subject of creativity in general and mathematical and geometric creativity in particular, we specified a grading method for the test. Through this method, each student will have 4 individual scores for fluency, flexibility, originality, and elaboration for each item of the test as well as an overall score of geometric creativity, as follows:

Fluency: The number of relevant responses. Each relevant response is given one point.

Flexibility: The number of different categories of relevant responses: answers, methods, or questions. Each flexibility category is given one point.

Originality/Novelty: It is the statistical infrequency of responses in relation to peer group. The more statistical infrequency the response has, the more originality it manifests. Each response is given zero, one, two, three or four points according to the following table:

Table 4

Originality scores for the geometric creativity test

[2] SeeEl-Rayashy & Ibrahim Al-Baz Mohamed 2000, Ibrahim Al-Baz Mohamed 1999, Lee & Shim 2005, Mohamed 2003.

The number of students who registered the response	1 Student	2 Student	3 Student	4 Student	5 Student
Originality score	4	3	2	1	0

Elaboration: It is graded by the number of follow-up questions or problems that are posed by redefining – substituting, combining, adapting, altering, expanding, eliminating, rearranging, or reversing – one or more aspects of the given geometric problem or situation. Each correct response is given one point.

Overall Geometric Creativity: It is the sum of fluency, flexibility, originality, and elaboration scores that represents the creative thinking ability in geometry.

2.5 Content Validity Check

For validating the GCT, the researchers presented it, in its preliminary form, to a group of judges specialized in teaching and learning mathematics in China, Egypt, and Germany. These judges reviewed the items, in their initial form, for clarity, readability, and appropriateness to measure what it is designed to measure and the level of mathematically gifted students in high schools.

Most changes suggested by the judges had to do with rhetorical and sequencing considerations. For one thing, upon the judges' request for the readability of the test items, the researchers used different fonts and font styles within the test items so that students could easily distinguish between the items statement and the items directions as well as quickly recognize the items tasks.

The judges also found that the question example given in item 3 is too complicated and it should be split into two questions. The question example was "Is it a plane figure such as a rectangle or a solid figure such as a sphere?" Thus, it was changed to: "Is it a plane figure such as a rectangle? Is it a solid figure such as a sphere?" For the same item, the judges recommended adding one more question, which is not Yes/No question. So, the researchers added one more question, which is "Does it have vertices?", "How many?" For item 9, the judges suggested changing the given example, which was "$\triangle$AEF and $\triangle$BDC is a pair of equivalent triangles" as it would restrict the students' thinking, causing them to only think about equivalent figures in terms of triangles. Accordingly, the researcher changed it to: "Triangle BCE and parallelogram ABDE is a pair of equivalent figures".

Finally, and more importantly, in items 5, 6, 7, 8, 9, 10, and 11 the judges were afraid that the mathematical symbols used in these items might not be recognized by the students in German schools as they use another system of symbols. For example: German students would not recognize $\overrightarrow{AM}$ as a ray; rather, it would be recognized as a vector. So we took care to use the same symbols used in German schools as shown in the German version[3].

[3] The German version of the geometric creativity test is available at:

http://cinderella.de/material/gct/

The English version of the geometric creativity test is available at:

http://cinderella.de/material/gct/

In conclusion, the judges were confident that the students being able to answer the test items show a certain degree of geometrical creativity, and vice-versa. They also asserted that the test items were appropriate to assess the assigned geometric components which they were designed for.

2.6 Test-Piloting

The researchers attempted a test piloting aiming at calculating: (1) the reliability coefficient for the test, (2) item-internal consistency reliability for the test items, (3) experimental validity for the test, and (4) the suitable time-range for the test. In this respect, the GCT was translated into German and administered to a sample of 30 students, 15 male and 15 female, at the University of Education Schwäbisch Gmünd at the end of the summer semester of the academic year 2008. Students' responses on the test were analyzed to calculate the scores of the geometric creativity components for each student.

2.6.1 The reliability coefficient

The reliability coefficient (Cronbach's α) for all test items as they measure geometric creativity was calculated using SPSS16. It was 0.83, a high reliability coefficient. Consequently, the GCT prepared by the researchers was proven reliable to measure the geometric creativity ability as a whole.

2.6.2 Item-internal consistency reliability

As for the item-internal consistency reliability, Cronbach's α is calculated for each of the geometric creativity component scores as subscales of the test, as follows:

For **fluency** the reliability coefficient was calculated for the fluency scores of the 12 items of the test as 0.62. To improve the reliability coefficient of the fluency component as a subscale of the test, SPSS suggested that if items 9 and 11 were deleted it might result in a better reliability coefficient for fluency. Indeed, deleting items 9 and 11 from the statistical analysis of the test items gave a reliability coefficient that equals 0.72, which is a good reliability coefficient. Consequently, measuring the fluency component of geometric creativity using items 1, 2, 3, 4, 5, 6, 7, 8, 10, and 12 of the prepared test can be considered reliable.

Regarding **flexibility** as a component of geometric creativity and a subscale of the test, the reliability coefficient was calculated for the flexibility scores of the 12 items of the test as 0.55. Again, to improve the reliability coefficient of the flexibility component as a subscale of the test, SPSS suggested that if item 9 and 11 were deleted it might result in a better reliability coefficient for the flexibility, and consequently deleting items 9 and 11 gave a reliability coefficient that equals 0.64, which is an acceptable reliability coefficient for this subscale. Consequently, measuring the flexibility component of geometric creativity using items 1, 2, 3, 4, 5, 6, 7, 8, 10, and 12 of the prepared test can be considered reliable.

As for **originality**, the reliability coefficient was calculated for the originality scores of the 12 items of the test as 0.59. Here, SPSS suggested that if item 11 was deleted it might result in a better reliability coefficient for the originality. The deletion of item 11 only improved the coefficient slightly to be 0.60, but again this can is an acceptable reliability coefficient for originality as a subscale of the test. Consequently, we consider measuring the originality component of geometric creativity using items 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 12 of the prepared test is reliable.

Regarding the **elaboration** component of the test, the reliability coefficient was calculated for the elaboration scores of items 5, 6, 10, and 11 that include the

elaboration component according to Table 2. We found it to be only 0.41. To improve the reliability coefficient of the elaboration component as a subscale of the test, SPSS also suggested that if item 11 was deleted it might result in a better reliability coefficient for the elaboration component. Anyway, even following this suggestion we found the reliability coefficient still at a low 0.51. Consequently, we do not have a reliable measure for the elaboration component, whether using items 5, 6, 10 and 11 or only items 5, 6 and 10.

Since the above-mentioned results suggest that items 9 and 11 should not be used for the total creativity measure, then we had to recalculate the overall reliability measures. After deleting items 9 and 11 from the statistical analysis of the test, the statistical attributes (mean, standard deviation, and Cronbach's α) of the overall geometric creativity test and its subscales (fluency, flexibility, originality, and elaboration) were re-calculated as shown in Table 5. The table shows that the subjects of the pilot test had a mean of 120.50 (SD = 42.08) and the reliability coefficient (Cronbach's α) for the geometric creativity test as a whole scale is 0.85 (high reliability coefficient) which means that the prepared geometric creativity test after deleting the two items can still be considered to be reliable to measure the geometric creativity ability as a whole.

Table 5

Statistics attributes of the pilot study of the geometric creativity test

Components of the geometric creativity	M	SD	Cronbach's α
Fluency	39.67	11.76	0.72
Flexibility	23.77	5.94	0.64
Originality	44.53	21.83	0.60
Elaboration	12.53	4.92	0.51
Overall Geometric creativity test	120.50	42.08	0.85

Regarding the geometric creativity component, **fluency**, the subjects had a mean of 39.67 (SD = 11.76) and the reliability coefficient was 0.72 (good reliability coefficient), which means that after deleting the two items, the prepared test is suitable for measuring the fluency component of geometric creativity.

As for the second geometric creativity component, **flexibility**, the subjects had a mean of 23.77 (SD = 5.94) and the reliability coefficient was 0.64 (accepted reliability coefficient), which means that after deleting the two items, the prepared test is suitable for measuring the flexibility component of geometric creativity.

Regarding the **originality** component, the subjects had a mean of 44.53 (SD = 21.83) and the reliability coefficient was 0.60 (accepted reliability coefficient), which means that after deleting the two items, the prepared test is suitable for measuring the originality component of geometric creativity.

Concerning the **elaboration** component, the subjects had a mean of 12.53 (SD = 4.92) and the reliability coefficient was 0.51 (low reliability coefficient). One interpretation for low consistency of elaboration component would be because the elaboration component of geometric creativity has many subscales (aspects) to measure, which have a negative effect on the consistency of the component items. Even though the reliability coefficient for the elaboration component was low, we believe that the elaboration construction is an important component of geometric creativity. However,

we cannot show that the GCT in its current form is able to measure it. These findings evoke the need for further studies with a bigger sample size of students to get more information about the test's reliability and its subscales.

2.6.3 Experimental validity

The experimental validity of the test as an estimation of the test validity was also calculated by taking the square root of the test reliability coefficient (Angoff, 1988, p. 20). It was calculated both before deleting items 9 and 11 as 0.913 and after deleting items 9 and 11 it was 0.922, which shows that the geometric creativity test has a high experimental validity.

2.6.4 Determining a suitable time-range

The time each subject took to finish the test was measured. Table 6 shows the statistical attributes of the time taken by the subjects in the pilot test. The subjects had a mean 94 (SD = 17.16), median = 90, mode = 85.

Table 6

Statistical attributes of the test time in the pilot testing

Mean	Median	Mode	SD	Minimum	Maximum
94	90	85	17.16	60	145

To determine the suitable time-range for the test, the researchers calculated the time each student tested took then calculated the mean of the time the first student took (60 minutes) and the last one took (145 minutes), so the suitable time of the test was calculated as approximately 100 minutes, as given by $\frac{60+145}{2} = 102.5$.

As items 9 and 11 do not contribute to calculating the reliability of the test as seen above, they might be omitted from the complete test in order to fit the necessary time-range to be 90 minutes.

3 Conclusion

In this paper we presented in detail the development of a geometric creativity test, which can be used to assess geometric creativity in terms of the adopted components (fluency, flexibility, originality, and elaboration). The findings of the pilot study, which are related to reliability of the test as well as the factor analysis of test subscales, evoke the need for further study to qualify the test in the light of these issues with bigger sample of students. We encourage other researchers to use the GCT in their own research, and appreciate any further data.

4 References

Angoff, W. H. (1988). Validity: An evolving concept. In H. Wainer and H. Braun (Eds.) *Test validity*. pp. 19-32. Lawrence Erlbaum Associates.

Cho, H., Han, H., Jin, M., Kim, H., & Song, M. (2004). Designing a micoworld: Activities and programs for gifted students and enhancing mathematical creativity. *Proceeding of the 10th conference of the International Congress on Mathematics Education, TSG 4: Activities and Programs for Gifted Students,* pp. 110-118*,* Copenhagen, Denmark, July 4-11

El-Demerdash, M. (in preparation). The effectiveness of an enrichment program using dynamic geometry software in developing mathematically gifted students' geometric creativity in high schools. Doctoral dissertation, University of Education Schwäbisch Gmünd, Germany.

El-Rayashy, H. A. M. & Ibrahim Al-Baz Mohamed, A. (2000). A proposed strategy on group mastery learning approach in developing geometric creativity and reducing problem solving anxiety among preparatory stage students. *Journal of Mathematics Education in Faculty of Education - Benha, Zagazig University, Vol. 3, July 2000,* pp. 65-207.

Haylock, D. (1997). Recognizing mathematical creativity in schoolchildren. *Zentralblatt für Didaktik der Mathematik, Vol. 29, No. 3,* pp. 68-74.

Ibrahim Al-Baz Mohamed, A. (1999). The relative effectiveness of a proposed strategy for teaching solid geometry and the De Bono strategy in developing geometric creativity and creative perceptions in secondary stage. In A. Rogerson (Ed.) *Proceeding of the International conference on mathematics education into the 21st century: societal challenges, issues and approaches, Vol. 1,* pp. 315-316, Cairo, Egypt, November 14-18.

Lee, K. S. & Shim, S. K. (2005). Assessing creativity in mathematical activity program. *Paper presented at ICMI-EARCOME-3-Third ICMI East Asia Regional Conference on Mathematics Education TSG 6: Assessment,* Shanghai, China, August 7-12

Jerwan, F. (2002). *Thinking education: Concepts and applications*. Dar Al-Fikr publishing and printing, Oman, Jordon

Mann, E. L. (2005). Mathematical creativity and school mathematics: Indicators of mathematical creativity in middle school students. Doctoral dissertation, Connecticut University, United States.

Mohamed, A. I. Z. (2003). Enrichment program in geometry for creative thinking development for talented students, in mathematics in the preparatory stage. Master thesis, Tanta University, Egypt.

Nakin, J. N. (2003). *Creativity and divergent thinking in geometry education*. Doctoral dissertation, South Africa University, South Africa.