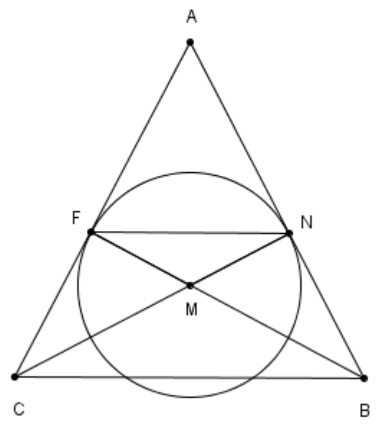
Item 8

In der nebenstehenden Zeichnung ist $\triangle ABC$ ein Dreieck. Die Strecke $\overline{AB}$ und die Strecke $\overline{AC}$ berühren den Kreis mit Mittelpunkt M in N und F, darüber hinaus schneiden sich die Strecken $\overline{BF}$ und $\overline{CN}$ in M.



Finde **möglichst viele Beweise dafür**, dass die beiden Dreiecke $\triangle ANF$ und $\triangle ABC$ ähnlich sind.

Wenn du mehr Platz brauchst, frage nach zusätzlichen Aufgabenblättern für diese Aufgabe.

Fluency: Each relevant response is given one point.

Flexibility: The number of different categories of relevant responses. Each flexibility category is given one point.

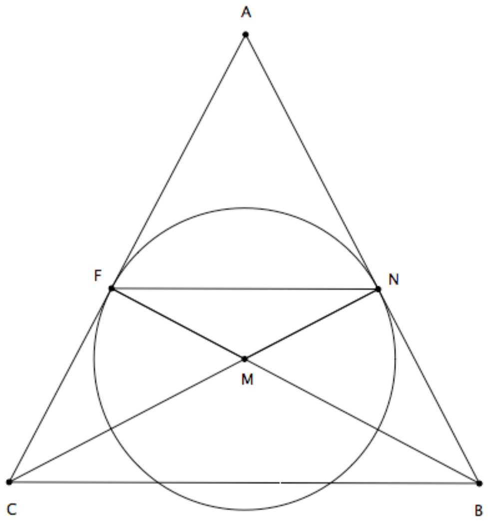
- C1 Responses that use equality of corresponding angles in the proof.
- C2 Responses that use proportionality of corresponding sides in the proof.

Originality/Novelty: It is the statistical infrequency of responses in relation to peer group. Each response is given zero, one, two, three or four points according to the following table:

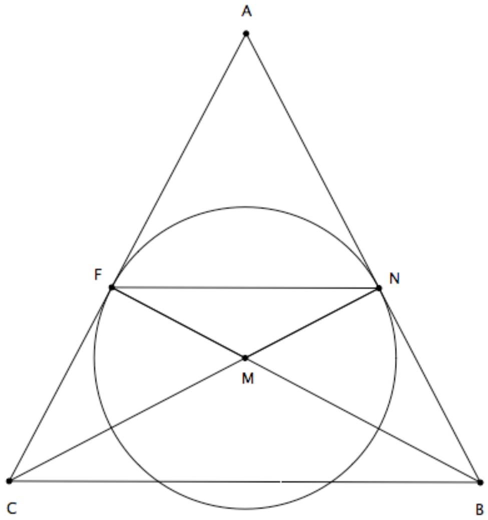
Grading originality points for the geometric creativity test

The number of students who registered the response	1 Student	2 Student	3 Student	4 Student	5 Student
Originality score	4	3	2	1	0

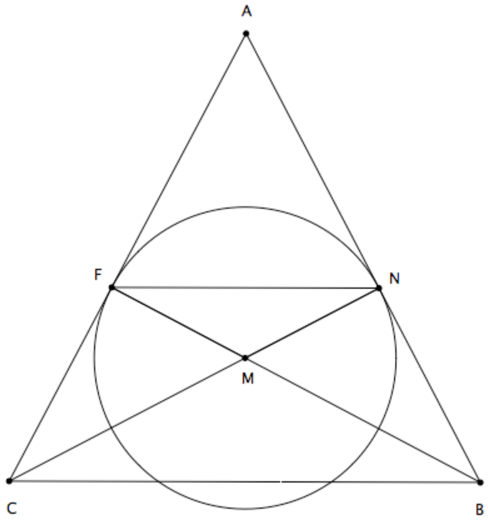
Student 1

Student's Responses	Flu.	Flex.	Ori.
 z. Z $|\measuredangle(AFN)| = |\measuredangle(ACB)|$, $|\measuredangle(FNA)| = |\measuredangle(CBA)|$ und $|\measuredangle(NAF)| = |\measuredangle(BAC)|$ (gilt sofort) Winkel $\measuredangle(AFN)$ und $\measuredangle(ACB)$ liegen beide auf dem Strahl AC^{+}. Sie sind genau dann gleich groß wenn $\overline{FN} \parallel \overline{CB}$. $\triangle AFM \equiv \triangle AMN$ (nach SSW: $|\measuredangle(MFA)| = |\measuredangle(MNA)| = 90°$; MF =MN Radius; MA gemeinsamer) … $|\overline{AF}| = |\overline{AN}|$ $\triangle CMF \equiv \triangle MNB$ (nach WSW: $|\measuredangle(CMF)| = |\measuredangle(BMC)|$ die Scheitelwinkel; MF =MN Radius;); $|\measuredangle(MFC)| = |\measuredangle(MNB)| = 90°$ … $|\overline{NB}| = |\overline{FC}|$ … $\frac{|\overline{AF}|}{|\overline{FC}|} = \frac{|\overline{AN}|}{|\overline{NB}|}$. Deswegen gilt und Umkehrung des Strahlensatz: $\overline{CB} \parallel \overline{FN}$ Also sind $|\measuredangle(NFA)| = |\measuredangle(BCA)|$ und $|\measuredangle(FNA)| = |\measuredangle(CBA)|$	1	C1	1
z. Z $\frac{|\overline{AF}|}{|\overline{AC}|} = \frac{|\overline{AN}|}{|\overline{AB}|} = \frac{|\overline{FN}|}{|\overline{CB}|}$ Nach den gleichen Argumentation wie oben gilt und Strahlensatz $\frac{|\overline{AF}|}{|\overline{FC}|} = \frac{|\overline{AN}|}{|\overline{NB}|}$ und nach? Strahlensatz $\frac{|\overline{AF}|}{|\overline{AC}|} = \frac{|\overline{FN}|}{|\overline{CB}|}$	1	C2	2
Score	2	2	3

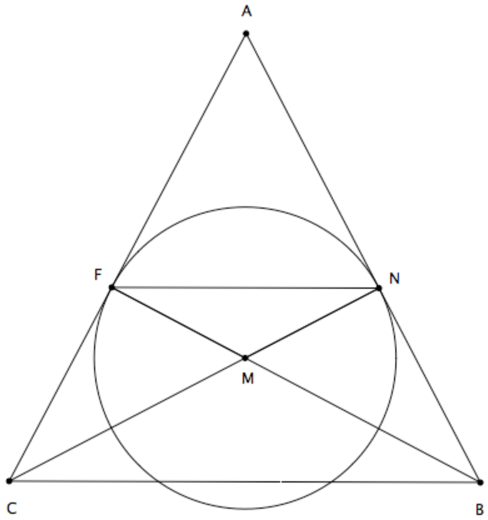
Student 2

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

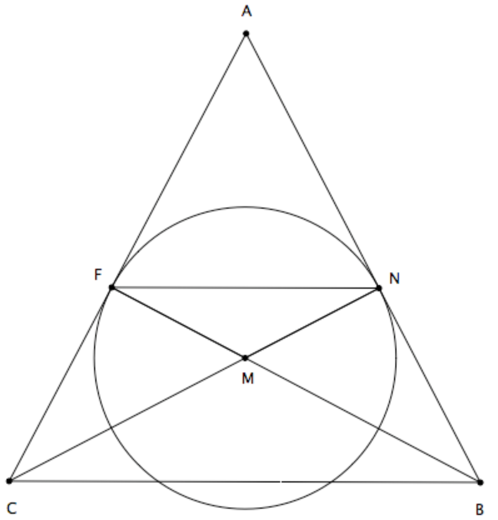
Student 3

Student's Responses	Flu.	Flex.	Ori.
			
No response	0	0	0
Score	0	0	0

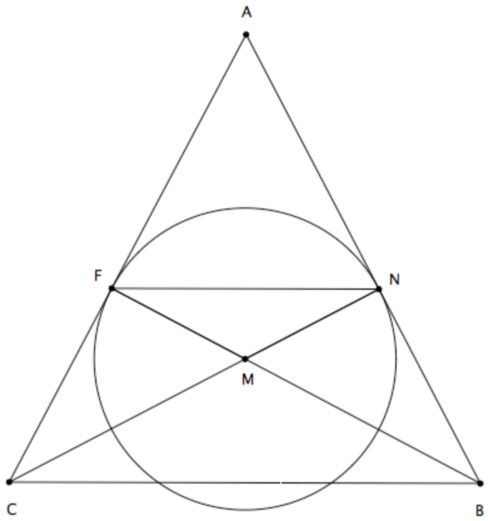
Student 4

Student's Responses	Flu.	Flex.	Ori.
 $|\measuredangle(CAB)| = |\measuredangle(FAN)|$, $|\overline{FM}|=|\overline{MN}|$ zwei Radien, $|\overline{MC}|=|\overline{MB}|$ … $\overline{CB} \parallel \overline{FN}$ Weil $|\overline{FB}|=|\overline{CN}|$ … $\frac{|\overline{AF}|}{|\overline{AC}|}=\frac{|\overline{AN}|}{|\overline{AB}|}=\frac{|\overline{FN}|}{|\overline{CB}|}$ … Streckenverhältnistreue	1	C2	2
γ bleibt gleich, α und α\` bzw. β und β\` sind Stufenwinkel, also gleich gros … Winkeltreue	1	C1	1
Score	2	2	3

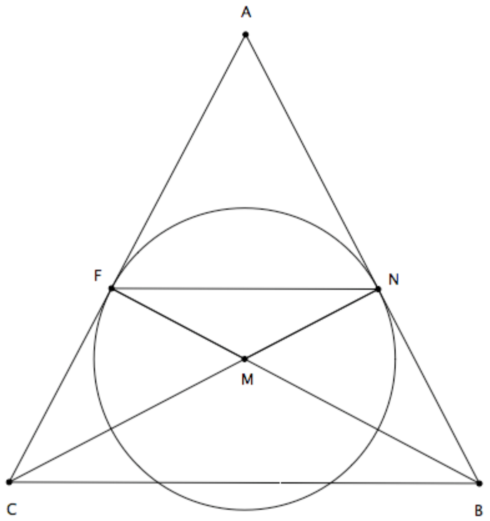
Student 5

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

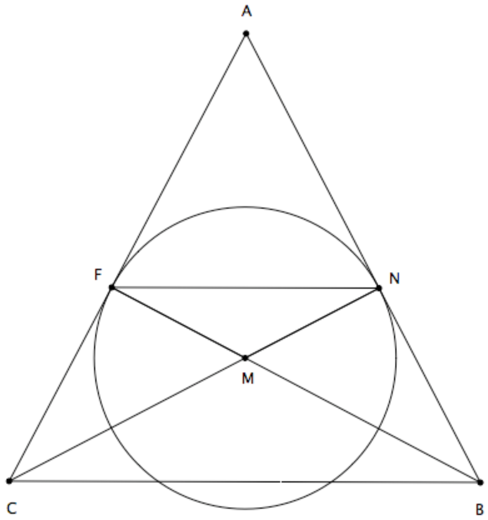
Student 6

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

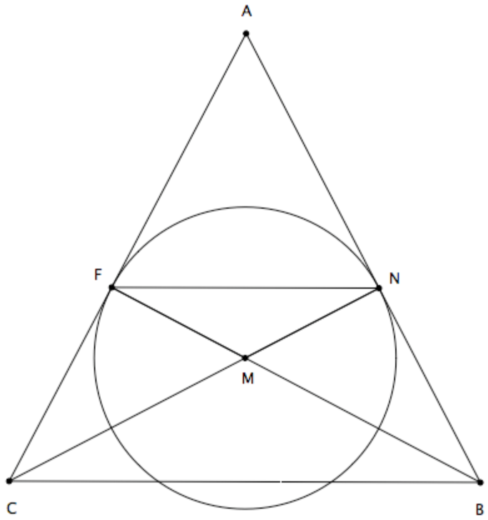
Student 7

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

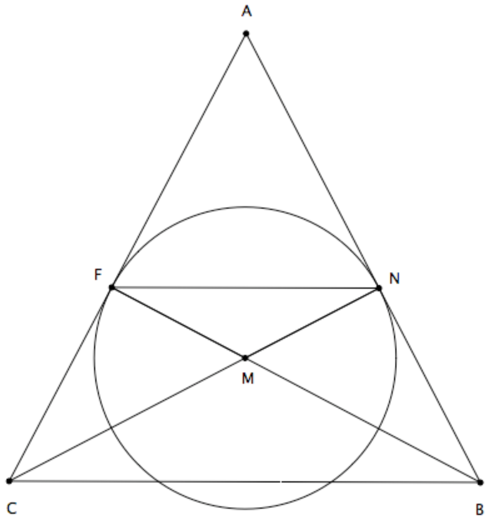
Student 8

Student's Responses	Flu.	Flex.	Ori.
 $\triangle CMF \equiv \triangle MBN$ (nach WSW) Also $\frac{\lvert\overline{AF}\rvert}{\lvert\overline{AC}\rvert} = \frac{\lvert\overline{AN}\rvert}{\lvert\overline{AB}\rvert}$ ... $\overline{FN} \parallel \overline{CB}$ Also $\alpha = \alpha$ $B_1 = B_2$ $\gamma_1 = \gamma_2$ ähnlich	1	C1	1
$\triangle MNA \equiv \triangle MBN$ (nach WSW) $\lvert\overline{CN}\rvert = \lvert\overline{BF}\rvert$ ... $\lvert\overline{FC}\rvert = \lvert\overline{NB}\rvert$ die Dreiecke sind ähnlich	1	C2	2
Score	2	2	3

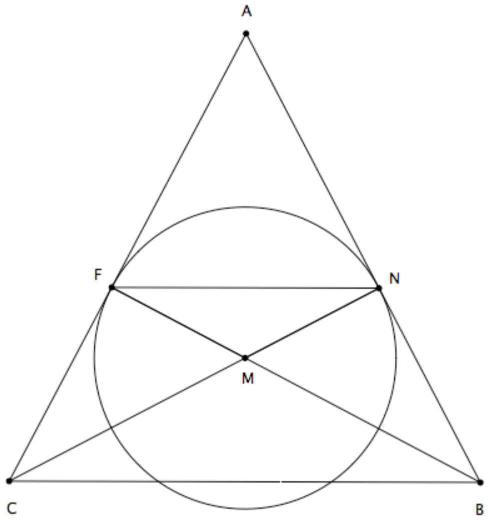
Student 9

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

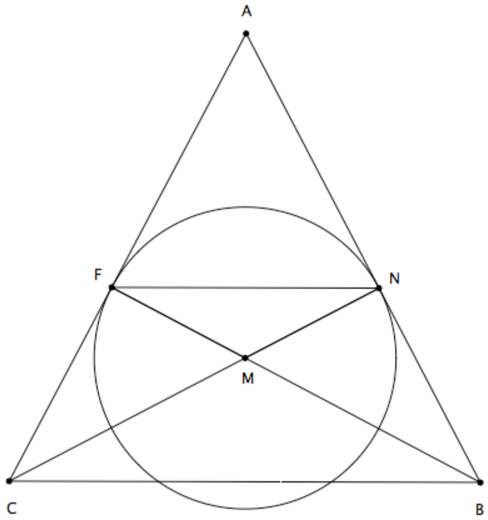
Student 10

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

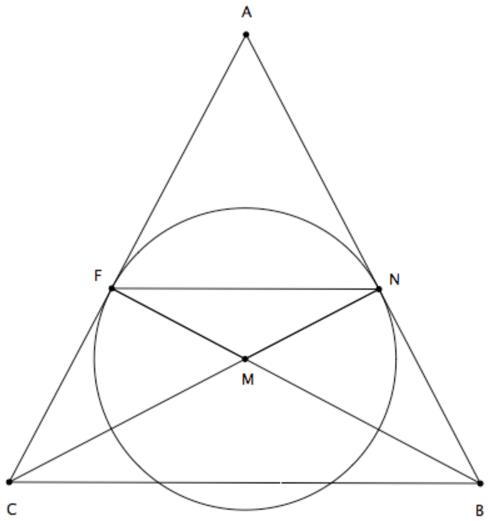
Student 11

Student’s Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

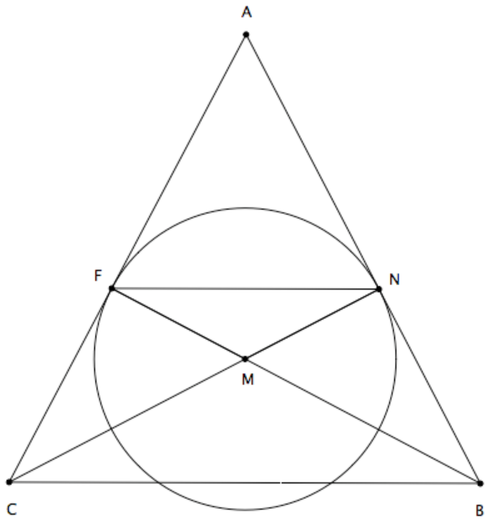
Student 12

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

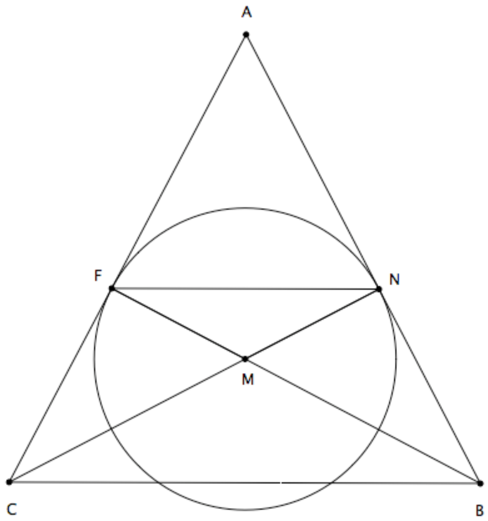
Student 13

Student’s Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

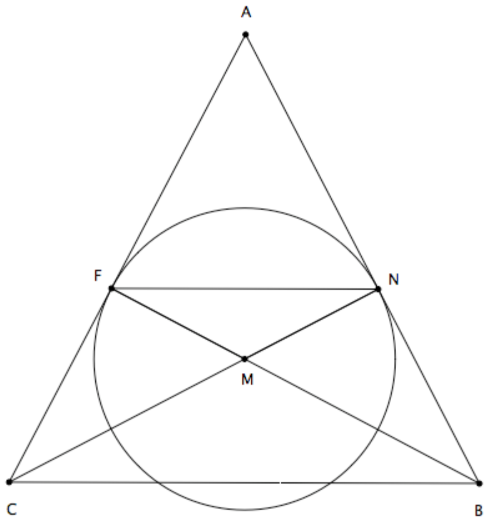
Student 14

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

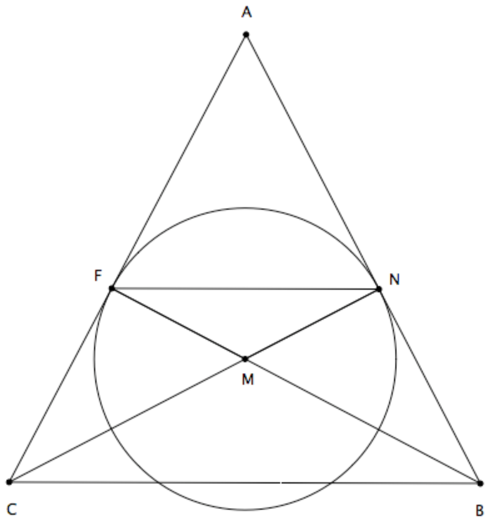
Student 15

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

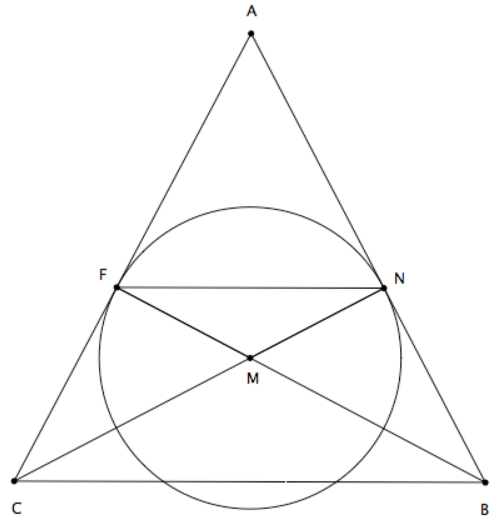
Student 16

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

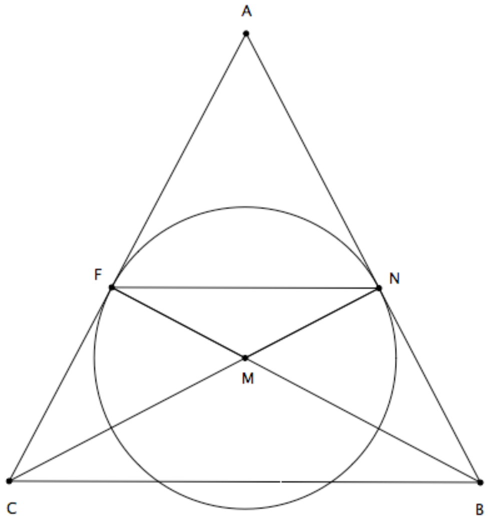
Student 17

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

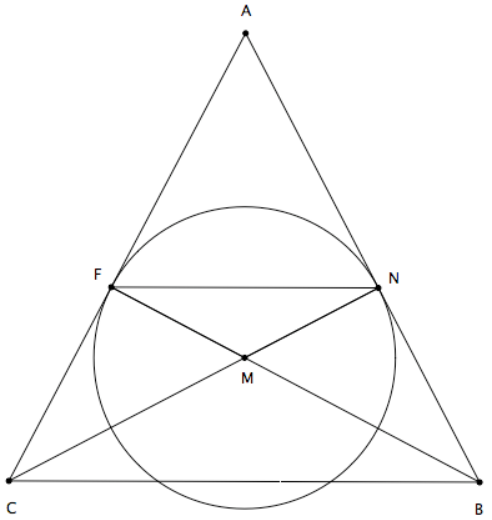
Student 18

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

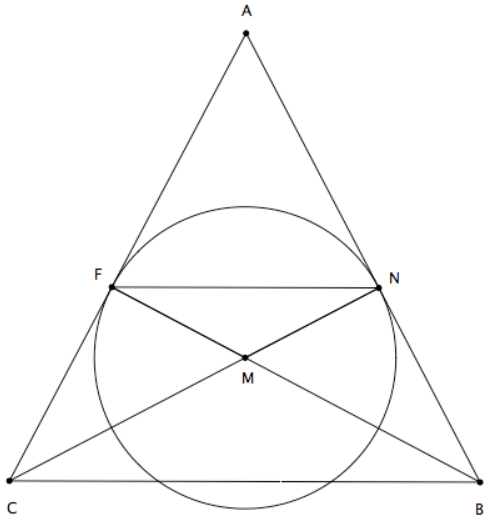
Student 19

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

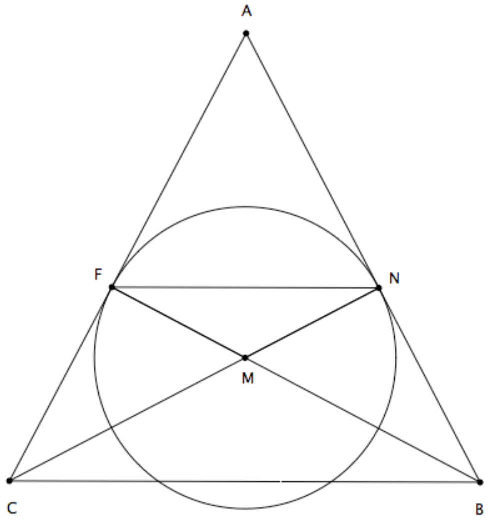
Student 20

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

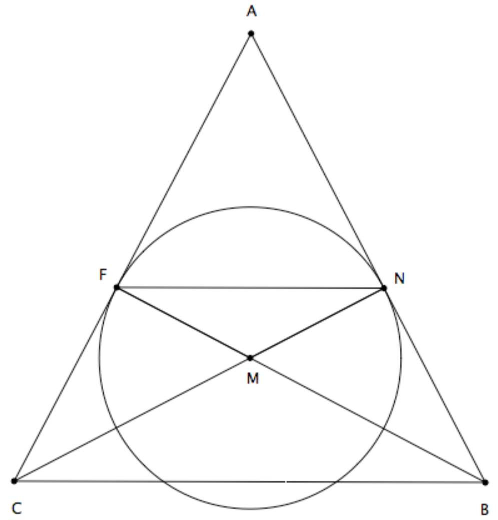
Student 21

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

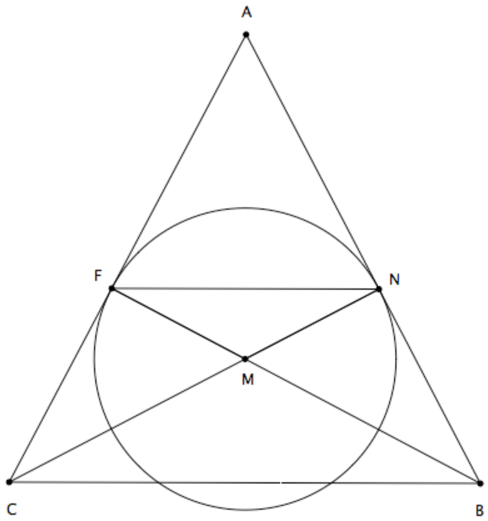
Student 22

Student's Responses	Flu.	Flex.	Ori.
 Bei beiden Dreiecken ist α gleich (haben ∡ gemeinsam). Wir müssen zeigen, dass $\overline{FN} \parallel \overline{CB}$, dann wären Dreiecke ähnlich. ...	1	C1	1
Score	1	1	1

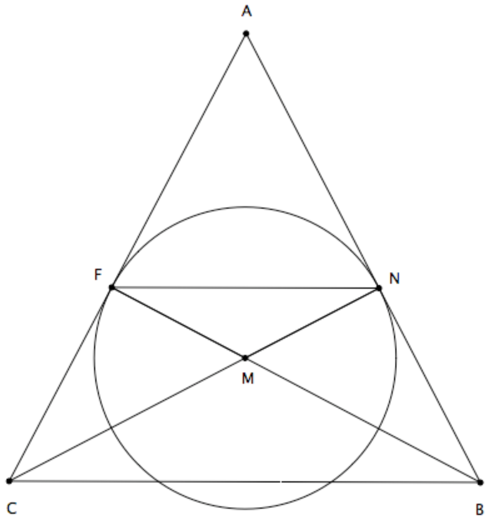
Student 23

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

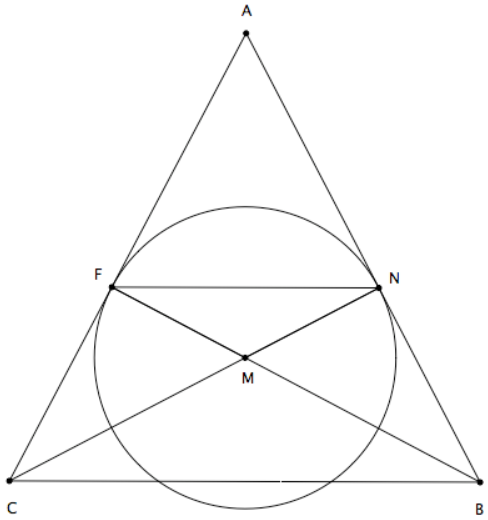
Student 24

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

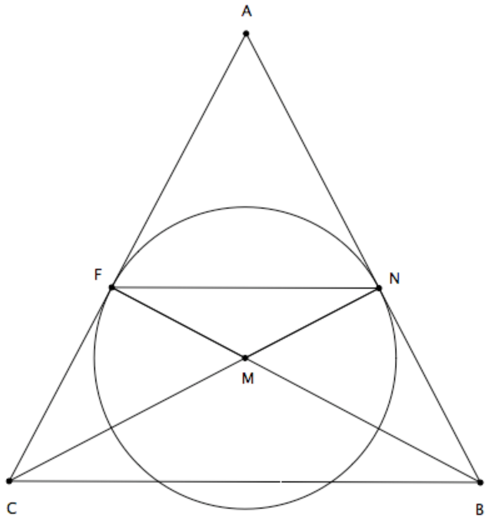
Student 25

Student's Responses	Flu.	Flex.	Ori.
			
Wrong response	0	0	0
Score	0	0	0

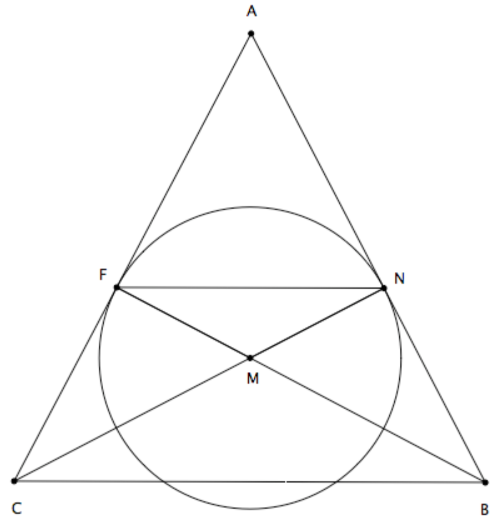
Student 26

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

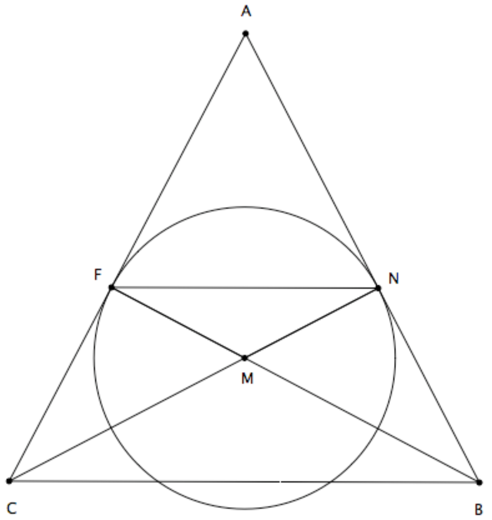
Student 27

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

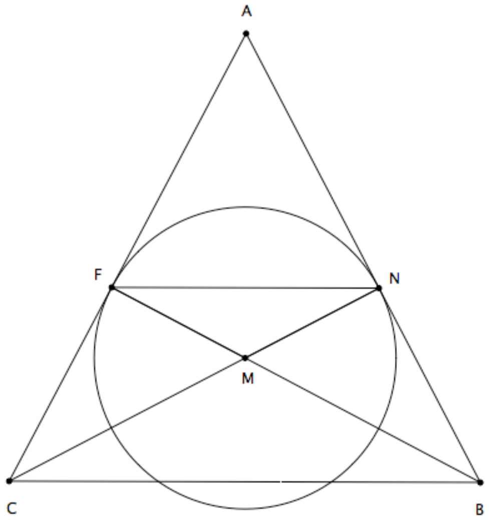
Student 28

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

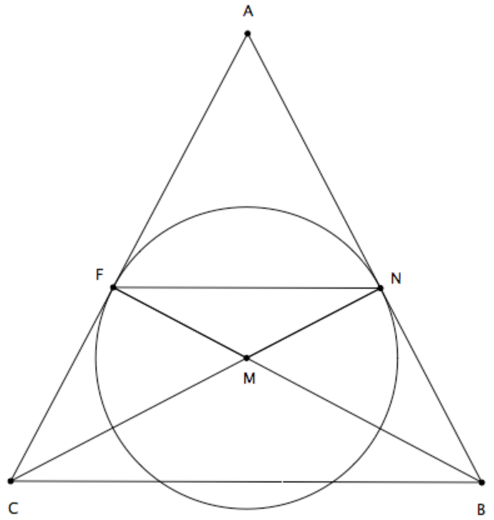
Student 29

Student's Responses	Flu.	Flex.	Ori.
			
Wrong response	0	0	0
Score	0	0	0

Student 30

Student's Responses	Flu.	Flex.	Ori.
 Wrong response	0	0	0
Score	0	0	0

Originality Scores for Students' Responses on Item 8

Student's Responses	Frequency	Originality Scores
 z. Z $\lvert\measuredangle(AFN)\rvert = \lvert\measuredangle(ACB)\rvert$, $\lvert\measuredangle(FNA)\rvert = \lvert\measuredangle(CBA)\rvert$ und $\lvert\measuredangle(NAF)\rvert = \lvert\measuredangle(BAC)\rvert$ (gilt sofort) Winkel $\measuredangle(AFN)$ und $\measuredangle(ACB)$ liegen beide auf dem Strahl AC^{+}. Sie sind genau dann gleich groß wenn $\overline{FN} \parallel \overline{CB}$. $\triangle AFM \equiv \triangle AMN$ (nach SSW: $\lvert\measuredangle(MFA)\rvert = \lvert\measuredangle(MNA)\rvert = 90°$; MF =MN Radius; MA gemeinsamer) … $\lvert\overline{AF}\rvert = \lvert\overline{AN}\rvert$ $\triangle CMF \equiv \triangle MNB$ (nach WSW: $\lvert\measuredangle(CMF)\rvert = \lvert\measuredangle(BMC)\rvert$ die Scheitelwinkel; MF =MN Radius;); $\lvert\measuredangle(MFC)\rvert = \lvert\measuredangle(MNB)\rvert = 90°$ … $\lvert\overline{NB}\rvert = \lvert\overline{FC}\rvert$ … $\frac{\lvert\overline{AF}\rvert}{\lvert\overline{FC}\rvert} = \frac{\lvert\overline{AN}\rvert}{\lvert\overline{NB}\rvert}$. Deswegen gilt und Umkehrung des Strahlensatz: $\overline{CB} \parallel \overline{FN}$ Also sind $\lvert\measuredangle(NFA)\rvert = \lvert\measuredangle(BCA)\rvert$ und $\lvert\measuredangle(FNA)\rvert = \lvert\measuredangle(CBA)\rvert$	4	1
z. Z $\frac{\lvert\overline{AF}\rvert}{\lvert\overline{AC}\rvert} = \frac{\lvert\overline{AN}\rvert}{\lvert\overline{AB}\rvert} = \frac{\lvert\overline{FN}\rvert}{\lvert\overline{CB}\rvert}$ Nach den gleichen Argumentation wie oben gilt und Strahlensatz $\frac{\lvert\overline{AF}\rvert}{\lvert\overline{FC}\rvert} = \frac{\lvert\overline{AN}\rvert}{\lvert\overline{NB}\rvert}$ und nach? Strahlensatz $\frac{\lvert\overline{AF}\rvert}{\lvert\overline{AC}\rvert} = \frac{\lvert\overline{FN}\rvert}{\lvert\overline{CB}\rvert}$	3	2